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Reconstruction of Five-Dimensional Bounce Cosmological Models from Deceleration Factor

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In this paper, we consider a class of five-dimensional Ricci-flat vacuum solutions, which contain two arbitrary functions $\mu(t)$ and $\nu(t)$. It is shown that $\mu(t)$ can be rewritten as a new arbitrary function $f(z)$ in terms of redshift z and the $f(z)$ can be determined by choosing particular deceleration parameters $q(z)$ which gives early deceleration and late time acceleration. In this way, the 5D cosmological model can be reconstructed and the evolution of the universe can be determined.

KEY WORDS: Kaluza–Klein theory; cosmology.

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1. INTRODUCTION

Recent observations of high redshift Type Ia supernovae reveal that our universe is undergoing an accelerated expansion rather than decelerated expansion (Barris *et al.*, 2003; Perlmutter *et al.*, 1999; Riess *et al.*, 1998; Tonry *et al.*, 2003; Know *et al.*, 2003). In addition, the discovery of Cosmic Microwave Background (CMB) anisotropy on degree scales together with the galaxy redshift surveys indicate $\Omega_{\text{total}} \simeq 1$ (de Bernardis *et al.*, 2000; Hanany *et al.*, 2000; Spergel *et al.*, 2000) and $\Omega_m \simeq 1/3$. All these results strongly suggest that the universe is permeated smoothly by 'dark energy,' which violates the strong energy condition with negative pressure. The dark energy and accelerating universe has been discussed extensively from different points of view (Armendáriz-Picón *et al.*, 1990; Caldwell, 2002; Caldwell *et al.*, 2003; Chiba, 2002; Hao and Li, 2003; Malquarti *et al.*, 2003; Sahni, 2002; Singh *et al.*, 2003; Steinhardt *et al.*, 1999; Turner, 2002; Zlatev *et al.*, 1999). In principle, a natural candidate to dark energy could be a small cosmological constant. However, there exist serious theoretical problems: fine tuning problem and coincidence problem. To overcome the fine tuning problem, some

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self-interact scaler fields ϕ with an equation of state (EOS) $w_\phi = p_\phi/\rho_\phi$ were introduced dubbed quintessence, where w_ϕ is time varying and negative. In nature, the potentials of the scalar field would be determined from the underlying physical theory, such as Supergravity, Superstring/M-theory etc.. However, disregarding these underlying physical theories just from the phenomenal level, we can design many kinds of potentials to solve the concrete problems (Sahni, 2003). Once the potentials are given, EOS w_ϕ of dark energy can be found. On the contrary, the potential can be reconstructed by a given EOS w_ϕ (Guo *et al.*, 2005). Then, the forms of the scalar potential are obtained by the observations data with given EOS w_ϕ. However, it is mentioned that the same models have early deceleration epoch. And, the acceleration of the universe just begins at not distance past. So, starting from the deceleration parameter, we can also reconstruct the cosmological models (Banerjee and Das *et al.*, 2005). This is the main motivation of this paper.

The idea that our world may have more than four dimensions is due to Kaluza (1921), who unified Einstein's theory of General Relativity with Maxwell's theory of Electromagnetism in a 5D manifold. In 1926, Klein reconsidered the idea and treated the extra dimension as a compact small circle topologically (Klein, 1926). However, up to now, there is no experimental evidence and indication for the existence of extra dimensions. Nevertheless, there is strong theoretical motivation for considering our world spacetime with more than three spatial dimension. M-theory is a higher dimensional theory with seven extra spatial dimensions, which try to incorporate quantum gravity in a consistent way. Recently, in higher dimension frame, brane theory has been studied extensively. In brane world theory, our four dimensional world is a hypersurface (brane) embedded in a higher dimensional manifold (bulk) and all forces are confined on the brane but gravity can propagate in the bulk. Also, it has interesting consequence in cosmology, for recent review seen (Maartens, 2004). Also, in higher dimensional frame, the Space-Time-Matter (STM) theory as a modern Kaluza-Klein theory is designed to incorporate the geometry and matter by Wesson and his collaborators (Wesson, 1999; Ponce de Leon, 2001). In STM theory, our world is hypersurface embedded in 5D Ricci-flat ($R_{AB} = 0$) manifold and all the matter in our world are induced from the higher dimension. The theory requires the metric components containing the extra dimension, saying y. And, the compactification or not of the extra dimension is not necessary in STM theory, for Ricci flat ($R_{AB} = 0$) five-dimensional manifold. The consequent cosmology in STM theory is studied in (Liu, 2002; Liu and Wesson, 2001, 2003; Ponde de Leon, 1988; Seahra and Wesson, 2003; Wang *et al.*, 2004; Xu *et al.*, 2003; Xu and Liu, 2004).

2. DARK ENERGY IN A CLASS OF 5D COSMOLOGICAL MODEL

Within the framework of STM theory, a class of exact 5D cosmological solution was given by Liu and Mashhoon (1995). Then, in (Liu and Wesson,

2001) restudied the solution and showed that it describes a cosmological model with a big bounce as opposed to a big bang. The 5D metric of this solution reads

$$dS^2 = B^2 dt^2 - A^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2 \tag{1}$$

where $d\Omega^2 \equiv (d\theta^2 + \sin^2 \theta d\phi^2)$ and

$$A^2 = (\mu^2 + k)y^2 + 2\nu y + \frac{\nu^2 + K}{\mu^2 + k},$$

$$B = \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}. \tag{2}$$

Here $\mu = \mu(t)$ and $\nu = \nu(t)$ are two arbitrary functions of t, k is the $3D$ curvature index ($k = \pm 1, 0$), and K is a constant. This solution satisfies the 5D vacuum equation $R_{AB} = 0$. So, the three invariants are

$$I_1 \equiv R = 0, I_2 \equiv R^{AB} R_{AB} = 0,$$

$$I_3 = R_{ABCD} R^{ABCD} = \frac{72K^2}{A^8}. \tag{3}$$

The invariant I_3 in (3) shows that K determines the curvature of the 5D manifold. It would be pointed out that the 5D and 4D Planck mass are not related directly, because of ${}^5G_{AB} = 0$, in the STM theory. So, in 4D we can take $\kappa_4^2 = 8\pi G_4$, where G_4 is 4D Newtonian gravitation constant.

Using the 4D part of the 5D metric (1) to calculate the 4D Einstein tensor, one obtains

$${}^{(4)}G^0_0 = \frac{3(\mu^2 + k)}{A^2},$$

$${}^{(4)}G^1_1 = {}^{(4)}G^2_2 = {}^{(4)} G^3_3 = \frac{2\mu\dot{\mu}}{A\dot{A}} + \frac{\mu^2 + k}{A^2}. \tag{4}$$

In our previous works (Wang *et al.*, 2004; Xu and Liu, 2004), the induced matter was set to be a conventional matter plus a time variable cosmological 'constant' or three components: dark matter radiation and x-matter. In Xu and Liu (2006), we pointed out the correspondence between the arbitrary function $f(z)$ and scalar field potentials. In this paper, we assume that the induced matter contains two parts, for simplicity: cold dark matter (CDM) ρ_{cd} and dark energy (DE) ρ_{de}. So, we have

$$\frac{3(\mu^2 + k)}{A^2} = \rho_{cd} + \rho_{de},$$

$$\frac{2\mu\dot{\mu}}{A\dot{A}} + \frac{\mu^2 + k}{A^2} = -\left(p_{cd} + p_{de}\right), \tag{5}$$

where

$$p_{cd} = 0, \quad p_{de} = w_{de}\rho_{de}. \tag{6}$$

From (5) and (6), one obtains the EOS of the dark energy

$$w_{de} = \frac{p_{de}}{\rho_{de}} = -\frac{2\mu\dot{\mu}/A\dot{A} + (\mu^2 + k)/A^2}{3(\mu^2 + k)/A^2 - \rho_{cd0}A^{-3}}, \tag{7}$$

and the dimensionless density parameters

$$\Omega_{cd} = \frac{\rho_{cd}}{\rho_{cd} + \rho_{de}} = \frac{\rho_{cd0}}{3(\mu^2 + k)A}, \tag{8}$$

$$\Omega_{de} = 1 - \Omega_{cd}. \tag{9}$$

where $\rho_{cd0} = \bar{\rho}_{cd0}A_0^3$, and Ω_{cd} and Ω_{de} are dimensionless density parameters of CDM and DE respectively. The Hubble parameter and deceleration parameter should be given as (Liu and Wesson, 2001; Wang and Lu, 2004; Xu and Liu, 2004),

$$H \equiv \frac{\dot{A}}{AB} = \frac{\mu}{A} \tag{10}$$

$$q\,(t, y) \equiv -A\frac{d^2A}{d\tau^2}\Bigg/\left(\frac{dA}{d\tau}\right)^2 = -\frac{A\dot{\mu}}{\mu\dot{A}}, \tag{11}$$

from which we see that $\dot{\mu}/\mu > 0$ represents an accelerating universe, $\dot{\mu}/\mu < 0$ represents a decelerating universe. So the function $\mu(t)$ plays a crucial role of defining the properties of the universe at late time. In this paper, we consider the spatially flat $k = 0$ cosmological model. From the above (7)–(11), it is easy to find that the equations does not contain $\nu(t)$ explicitly, which is included in A. So, to avoid boring choice of the concrete forms $\nu(t)$, we use $A_0/A = 1 + z$ and define $\mu_0^2/\mu_z^2 = f(z)$, and then find that the (7)–(11) can be reduced to as follows in redshift z

$$w_{de} = -\frac{1 + (1 + z)\,d\ln f(z)/dz}{3 - 3\Omega_{cd}}, \tag{12}$$

$$\Omega_{cd} = \Omega_{cd0}\,(1 + z)\,f(z), \tag{13}$$

$$\Omega_{de} = 1 - \Omega_{cd}, \tag{14}$$

$$q = \frac{1 + 3\Omega_{de}w_x}{2} = -\frac{(1 + z)}{2}\frac{d\ln f(z)}{dz}. \tag{15}$$

From (15), we find that it is an ordinary differential equation of function $f(z)$ w.r.t. redshift z, once one form of $q(z)$ is given. In (Banerjee *et al.*, 2005), the scalar potentials are constructed from a given deceleration parameter $q = -1 - pa^p/(1 + a^p)$, where p is a constant. In our case, with this spirit, we also can reconstruct the forms of function $f(z)$ from a given concrete form of $q(z)$. As an example, we consider the following deceleration parameter

$$q(z) = \frac{1}{2} - \frac{\alpha}{(1+z)^{\beta}}, \tag{16}$$

where, α and β are positive constants determined by observation values. By this assumption, one obtains $f(z)$

$$f(z) = \frac{C \exp\left(-\frac{2\alpha(1+z)^{-\beta}}{\beta}\right)}{1+z}, \tag{17}$$

where, $C = \exp(2\alpha/\beta)$ is an integral constant determined by $f(0) \equiv 1$. Once $f(z)$ is specified, the evolution parameters, such as Ω_{cd} Ω_{de} and w_{de} are obtained at one time. From (16), one finds that

$$q_0 = \frac{1}{2} - \alpha. \tag{18}$$

The current observation value $q_0 = -0.67 \pm 0.25$ determines the parameter α. And, the parameter β has relation with the ratio of Ω_{cd} and Ω_{de}, which is constrained by early cosmological observations. By setting $\alpha = 1.17$ and $\beta = 1.7$, the evolution of these parameters is plotted in Fig. 1.

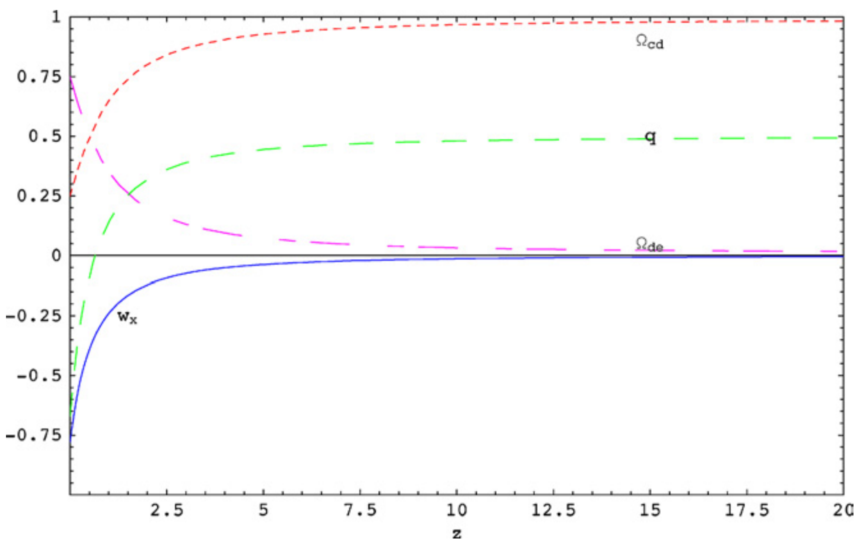

Fig. 1. The evolution of the dimensionless density parameters Ω_{cd}, Ω_{de}, and deceleration parameter q, EOS of dark energy w_{de} versus redshift z, where $\Omega_{cd0} = 0.25$, $\Omega_{de0} = 0.75$, $\alpha = 1.17$ and $\beta = 1.7$.

3. CONCLUSIONS

A general class of 5D cosmological models is characterized by a big bounce as opposed to the big bang in 4D standard cosmological model. This exact solution contains two arbitrary functions $\mu(t)$ and $\nu(t)$. By careful observation, one finds that the dimensionless density, deceleration parameters and EOS of dark energy are independent on $\nu(t)$ explicitly which is included in A. In terms of redshif z, $\mu(t)$ can be rewritten into a new arbitrary function $f(z)$. Once the forms of the arbitrary function $f(z)$ are specified, the universe evolution will be determined. In this paper, we reconstruct the arbitrary function $f(z)$ by assuming a particular form of deceleration parameter $q(z)$. In this way, the 5D cosmological models are reconstructed and the evolution of the universe can be determined.

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